Heiko Gimperlein

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Edinburgh, EH14 4AS

Research Interests

Numerical analysis and partial differential equations, in particular:

- Pseudodifferential and boundary integral operators, microlocal analysis
- Theoretical numerical analysis of boundary problems
- Applications in engineering, computer science and the natural sciences

Education

Leibniz University Hannover, Hannover/Germany

Ph.D. in Mathematics (2010)
Thesis title: *Topics in singular analysis with applications to numerical analysis and to representation theory.*

Diploma in Mathematics (2006), Diploma in Physics (2005)

Employment

Maxwell Institute for Mathematical Sciences and Heriot–Watt University, Edinburgh
Associate professor / Reader (2016 –)
Assistant professor / Lecturer (2013 – 2016, permanent)

Visiting research professor, University of Paderborn
(May 2015 – May 2016, April – July 2017, Nov – Dec 2019)

Postdoc, University of Copenhagen (July 2010 – July 2013)

Research assistant, Leibniz University Hannover (Oct 2006 – June 2010)

Awards

Wissenschaftspreis Hannover 2010 (biennial thesis award of Leibniz University)

Prediploma Award of the Christian Kuhlemann Foundation and Leibniz University

Fellowship of the German National Merit Foundation

Students

Ph.D.: A. Alpyspayeva, N. Louca, K. Quaine, R. He and D. Torkington (with A. A. Lacey)

graduated Ph.D. students:
J. Stocek (→ postdoc Cambridge / British Antarctic Survey)
C. Özdemir (with E.P. Stephan → postdoc TU Graz)
G. Estrada-Rodriguez (→ FSMP fellow and postdoc, Sorbonne, Paris)
M. Iqbal (with O. Laghrouche / M.S. Mohamed → permanent researcher, National University of Science and Technology, Pakistan)
D. Stark (→ Canon medical)

5 M.Sc. students, 1 M.Math. student, 11 B.Sc. students, 8 undergraduate research projects

Extended Research Visits

University of Paderborn *(visiting research professor)*	April–July 2017, Nov–Dec 2019
Institut Henri Poincaré / Centre Émile Borel, Paris	Oct 2016
Technical University of Vienna *(visiting professor)*	June 2016
University of Paderborn *(visiting research professor)*	May 2015 – May 2016
École Normale Supérieure, Paris *(professeur invité)*	Nov / Dec 2014
Centre de Recerca Matemàtica, Barcelona	June / July 2013
Rutgers, New Brunswick	Nov 2012
Max Planck Institute for Mathematics, Bonn *(postdoc)*	Aug / Sept 2011
Institut Henri Poincaré / Centre Émile Borel, Paris	June 2009
Mathematical Sciences Research Institute, Berkeley	Sept – Dec 2008
ETH Zürich	June / July 2008

Lecturing Visits

African Institute for Mathematical Sciences, Biriwa	May 2014

Heiko Gimperlein

Publications

36 refereed articles (as listed in Web of Science), in numerical analysis, pure and applied analysis, representation theory and computational engineering

Invited Talks (since 2010)

More than 50 invited talks at international mathematics and engineering conferences.

More than 45 invited talks in departmental seminars since 2010, e.g. in Bonn, Oxford, Paris, Stanford.

Grants

Total of ca. €1.400.000, including:

Danish Science Foundation FNU postdoctoral grant (2010–2012)

EPSRC Impact Acceleration Award (2016–2017)

Funding for Ph.D. students: Avicenna Foundation (Özdemir), Bolashak presidential scholarship (Alpyspayeva), Chinese funding (He), EPSRC CDT (Estrada, Louca, Stark, Stocek), James-Watt scholarship (Iqbal), AWE plc. (Quaine, Torkington).

Numerous smaller grants for research beyond departmental resources, e.g. from Clay Institute, Scottish Funding Council, London Mathematical Society.

Conference Organization

13 international conferences and 4 graduate schools, most recently:

Convex Integration and Nonlinear PDEs
(with G.-Q. Chen, R.J. Knops, M. Slemrod, L. Szekelyhidi, ICMS / Edinburgh, 2021)

Mathematical Modelling of Energetic Materials
(with A.A. Lacey, J. Curtis, ICMS / Edinburgh, 2019)

Magnitude 2019 – Analysis, Category Theory, Applications
(with M. Goffeng, T. Leinster, Edinburgh, 2019)

Recent Advances in Enriched Finite and Boundary Element Methods
Minisymposium within *European Conference on Computational Mechanics*
(with O. Laghrouche, M.S. Mohamed, Glasgow, 2018)

Nonlinear Analysis and the Physical and Biological Sciences
(with J.M. Ball, J.C. Eilbeck, M. Grinfeld, R.J. Knops, ICMS / Edinburgh, 2018)

Activities and Service

Organizer of Analysis seminar (2014 – 2015, since 2016)

Maxwell Mini-Symposia Analysis and its Applications, resp. PDE (since Feb 2015)

Theme head for pure/applied analysis in Scottish Mathematical Sciences Training Centre

Reviewer for grant proposals: EPSRC (UK), European Science Foundation, FWO (Belgium), FWF (Austria), GACR (Czech Republic), MIUR (Italy)

Reviewer for numerous journals in numerical analysis, pure and applied analysis, acoustics and computational engineering

Department representative on
North British Functional Analysis Seminar (since June 2014),
Edinburgh Mathematical Society General Committee (2013 – 2016).

Acting Director for Training of the EPSRC Centre for Doctoral Training MIGSAA (2017, deputy 2015 – 2016)

Member of Mathematics Undergraduate Board of Studies (2014 – 2015)

Teaching

M.Sc. course *Research and Industry Topics*	Spring 2020
Ph.D. course *Variational Methods for PDEs* (with J.M. Ball)	Spring 2019
Ph.D. course *Numerical Analysis*	Spring 2018 / 20
Ph.D. course *Harmonic Analysis and Function Spaces* (broadcast via SMSTC)	Spring 2017
Ph.D. short course *Interface and Contact Problems* (TU Wien)	June 2016
Ph.D. course *Pure Analysis 2* (1/3 of course, broadcast, SMSTC)	Spring 2015/17/18/19/20
Ph.D. course *Applied Analysis and PDEs 2* (broadcast, SMSTC)	Spring 2015/17/18/19/20
B.Sc. course *Numerical Analysis A*	Spring 2014 / 15
M.Sc./Ph.D. course *Pseudodifferential Operators and Spectral Theory*	Fall/Winter 2011
M.Sc. course *Differential Operators and Function Spaces II*	Summer 2011 / 12
B.Sc. course *Introduction to Partial Differential Equations*	Fall 2010

Significant Publications

1) with M. Goffeng, *On the magnitude function of domains in Euclidean space*, American Journal of Mathematics, to appear (2021).
(+ 4 invited blog posts at n-Category Cafe, https://golem.ph.utexas.edu/category)

2) with J. Stocek, C. Urzua-Torres, *Optimal operator preconditioning for pseudodifferential boundary problems*, Numerische Mathematik, to appear (2021).

3) with B. Krötz, *On Sobolev norms for Lie group representations*, Journal of Functional Analysis 280 (2021), 108882.

4) with C. Özdemir, D. Stark and E. P. Stephan, *A residual a posteriori error estimate for the time-domain boundary element method*, Numerische Mathematik 146 (2020), 239 – 280.

5) with G. Estrada-Rodriguez and E. Estrada, *Metaplex networks: Influence of the exo-endo structure of complex systems on diffusion,* SIAM Review 62 (2020), 617 – 645, Research Spotlights.

6) with G. Estrada-Rodriguez, K. J. Painter and J. Stocek, *Space-time fractional diffusion in cell movement models with delay,* Mathematical Models and Methods in Applied Sciences 29 (2019), 65 – 88.

7) with F. Meyer, C. Özdemir, D. Stark and E. P. Stephan, *Boundary elements with mesh refinements for the wave equation*, Numerische Mathematik 139 (2018), 867 – 912.

8) with L. Banz, A. Issaoui and E. P. Stephan, *Stabilized mixed hp-BEM for frictional contact problems in linear elasticity*, Numerische Mathematik 135 (2017), 217 – 263.

9) with A. Costea and E. P. Stephan, *A Nash–Hörmander iteration and boundary elements for the Molodensky problem,* Numerische Mathematik 127 (2014), 1 – 34.

10) with B. Krötz and C. Lienau, *Analytic factorization of Lie group representations*, Journal of Functional Analysis 262 (2012), 667 – 681.

Research program "Time domain boundary elements for interface and free boundary problems"

Background: The study of elastodynamics has applications from mechanical and civil engineering to seismic risk assessment and geological soil analysis. The Finite Element Method is the main approach for its solution. However, for problems in unbounded domains, such as scattering problems, it requires to truncate the computational domain and impose suitable conditions on the artificial boundary. Reflections at this artificial boundary can invalidate the results. The Boundary Element Method (BEM) provides an efficient solver in unbounded domains, by reducing the problem to an integral equation on the bounded scatterer.

The BEM has become a standard tool for acoustic and elastic problems in the frequency domain. Directly in the time domain, BEM has attracted much recent interest, including for problems which are not feasible in the frequency domain due to nonlinearities or because they involve a wide range of frequencies. For the time-dependent problem, additional care is required to obtain numerically stable formulations, see [10, 19]. In particular, for the acoustic wave equation long-time stability and excellent approximation properties are known for weak formulations related to the energy [2, 11, 13, 14, 15].

Current research with A. Aimi and G. Di Credico from Parma initiates the study of high-order boundary elements for scattering problems for the time dependent Lamé equation in elasticity [6]. To be specific, it focuses on soft scattering problems, i.e. problems in the exterior $\mathbb{R}^n \setminus \overline{\Omega}$ of a polyhedral domain Ω, where $n = 2$ or 3, with Dirichlet boundary conditions. The problem is formulated as the equivalent weakly singular integral equation

$$\mathbf{f}(\mathbf{x},t) = \mathcal{V}\boldsymbol{\phi}(\mathbf{x},t) = \int_0^t \int_{\partial\Omega} G(\mathbf{x},\mathbf{y},t,s)\boldsymbol{\phi}(\mathbf{y},s)\; d\sigma_{\mathbf{y}}\; ds\,, \quad \text{for } (\mathbf{x},t) \in \partial\Omega \times [0,T]. \tag{1}$$

Here, $G = \{G_{ij}\}_{i,j=1}^n$ is the fundamental solution of the Lamé equation. Based on a careful regularity analysis of the solution $\boldsymbol{\phi}$ to (1), we develop discretizations which provably converge with optimal convergence rate to $\boldsymbol{\phi}$. They are based on geometrically graded meshes and may use polynomials of arbitrarily high degree (h, p and hp versions) [11, 13].

This project aims to use these recent theoretical advances for the Dirichlet and Neumann problems to study boundary element and coupled finite element/boundary element methods for interface problems and free boundary problems. Fundamental examples are the dynamic transmission problems, fluid-structure interaction and hyperbolic variational inequalities like the obstacle/contact problems in elasticity [12, 16], nonlinear problems which cannot be solved in frequency domain. While there has been much recent progress for the scalar wave equation [1, 3, 4, 7], several of the techniques rely on its scalar nature. The analysis and numerical analysis of key problems, in particular fluid-structure interaction and variational inequalities [17], are wide open and shall be investigated during this stay.

Goals: The analysis in [6] for the weakly singular operator $\mathcal{V}$ for the Dirichlet problem and the hypersingular operator associated to the Neumann problem provides a basis to consider the full Calderón projector for the time-dependent Lamé equation. A subtle question is the coercivity of this operator, which arises in the boundary integral formulation of most interface problems. In a first step, the analysis in [6] is extended to this setting.

Second, we use the coercivity of the Calderón projector to formulate a boundary element method for the classical Signorini (thin obstacle) problem for elasticity. The analysis and development of stable discretizations for this contact problem is a long-standing challenge [12, 16, 17], and even the existence of solutions is unknown. [12] provided the first provably convergent discretization for a scalar toy problem, by exploiting refined coercivity properties of the Dirichlet-Neumann operator for the wave equation which are not known for the Lamé equation. Nevertheless, initial work with Aimi, Di Credico and Stephan indicates the stability of a similar method also for the Lamé equation in $2d$. These preliminary results lead us to investigate the proposed method numerically and rigorously establish its stability. We might hope to prove convergence (and thereby existence of solutions) in special cases.

Third, we consider the coupling of finite elements and boundary elements. The coercivity of the Calderón projector allows to formulate stable formulations for interface problems involving two different media, like transmission or fluid-structure interaction. Starting from the joint work [6], we consider fast h, p and hp discretizations which resolve the singularities of the solutions from the interface and provably converge with optimal convergence rate. Key for the extension of the results from [6] to interface problems is the fine analysis of the regularity of solutions in [18].

While the three goals above focus on the analytical and approximation-theoretic problems, efficient codes require careful algorithmic considerations. Galerkin solutions to (1) based on piecewise constant ansatz and test functions in time lead to a linear system with block lower triangular Toeplitz matrix. The lower triangular form allows to solve the space-time system by backsubstitution, resulting in a *marching in on time* time stepping scheme. Because of the Toeplitz structure, only one spatial matrix needs to be computed and stored for each time step. Unfortunately, the matrix elements are space-time double integrals with weakly singular kernels, depending on primary and secondary wave speeds, multiplied by Heaviside functions modeling wave front propagation and functions not regular in the first spatial derivative. An efficient composite graded quadrature has been implemented in [5], and the analysis of this quadrature in 2D and 3D will be considered in this project.

References

[1] T. Abboud, P. Joly, J. Rodriguez, I. Terrasse, *Coupling discontinuous Galerkin methods and retarded potentials for transient wave propagation on unbounded domains*, J. Comp. Phys. 230 (2011), 5877–5907.

[2] A. Aimi, M. Diligenti, C. Guardasoni, I. Mazzieri, S. Panizzi, *An energy approach to space-time Galerkin BEM for wave propagation problems*, Internat. J. Numer. Methods Engrg. 80 (9) (2009), 1196–1240.

[3] A. Aimi, M. Diligenti, A. Frangi, C. Guardasoni, *Energetic BEM-FEM coupling for wave propagation in 3D multidomains*, Internat. J. Numer. Methods Engrg. 97 (2014), 377–394.

[4] A. Aimi, M. Diligenti, C. Guardasoni, S. Panizzi, *Energetic BEM-FEM coupling for wave propagation in layered media*, Commun. Appl. Ind. Math. 3 (2012), 418–438.

[5] A. Aimi, L. Desiderio, M. Diligenti, C. Guardasoni, *Application of Energetic BEM to 2D Elastodynamic Soft Scattering Problems*, Commun. Appl. Ind. Math. 10 (1) (2019) 182–198.

[6] A. Aimi, G. Di Credico, H. Gimperlein, E. P. Stephan, *h and hp time domain boundary elements for the Lamé equation*, in preparation (2021).

[7] L. Banjai, C. Lubich, F.-J. Sayas, *Stable numerical coupling of exterior and interior problems for the wave equation*, Numer. Math. 129 (2015), 611–646.

[8] L. Banz, H. Gimperlein, A. Issaoui, E. P. Stephan, *Stabilized mixed hp-BEM for frictional contact problems in linear elasticity*, Numer. Math. 135 (2017), 217–263.

[9] E. Bécache, T. Ha Duong, *A space-time variational formulation for the boundary integral equation in a 2d elastic crack problem*, ESAIM: Mathematical Modelling and Numerical Analysis 28 (2) (1994) 141–176.

[10] M. Costabel, F.-J. Sayas, *Time-Dependent Problems with the Boundary Integral Equation Method*, In Encyclopedia of Computational Mechanics Second Edition (eds E. Stein, R. Borst and T.J.R. Hughes). doi:10.1002/9781119176817.ecm2022 .

[11] H. Gimperlein, F. Meyer, C. Özdemir, D. Stark, E. P. Stephan, *Boundary elements with mesh refinements for the wave equation*, Numer. Math. 139 (2018), 867–912.

[12] H. Gimperlein, F. Meyer, C. Özdemir, E. P. Stephan, *Time domain boundary elements for dynamic contact problems*, Comp. Methods Appl. Mech. Engrg. 333 (2018), 147–175.

[13] H. Gimperlein, C. Özdemir, D. Stark, E. P. Stephan, *hp-version time domain boundary elements for the wave equation on quasi-uniform meshes*, Comp. Methods Appl. Mech. Engrg. 356 (2019), 145–174.

[14] H. Gimperlein, C. Özdemir, D. Stark, E. P. Stephan, *A residual a posteriori estimate for the time domain boundary element method*, Numer. Math. 146 (2020), 239–280.
[15] T. Ha Duong, A. Bamberger, *Formulation variationnelle espace-temps pour le calcul par potentiel retardé dela diffraction d'une onde acoustique (I)*, Math. Methods Appl. Sci. 8 (1) (1986) 405–435.
[16] P. Le Tallec, P. Hauret, B. Wohlmuth, C. Hager, *Solving dynamic contact problems with local refinement in space and time*, Comp. Methods Appl. Mech. Engrg. 201-204 (2012), 25–41.
[17] G. Lebeau, M. Schatzman, *A wave problem in a half-space with a unilateral constraint at the boundary*, J. Differential Equations 53 (1984), 309–361.
[18] S. I. Matyukevich, B. A. Plamenevskii, *On dynamic problems in the theory of elasticity in domains with edges*, Algebra i Analiz 18 (2006), 158–233.
[19] F.-J. Sayas, *Retarded Potentials and Time Domain Boundary Integral Equations: A Road Map*, Springer Series in Computational Mathematics 50, Springer, 2016

Course:	**Numerical Methods for Differential and Integral Equations**
Degree:	Mathematics
Type of course:	Supplementary subjects
Language of instruction:	English
Lecturers:	Gimperlein, Heiko
Academic year:	2021/2022
Semester:	Second semester (preferred)
Number of credits:	6
Unit coordinator:	Gimperlein, Heiko
Contact hours:	48
Individual work hours:	90

Learning outcomes of unit:

- Knowledge and understanding of elementary concepts for the numerical modeling of elliptic and parabolic partial differential equations, in particular, based on finite difference, finite element, spectral and boundary element methods.
- Ability to program the discussed numerical methods in Matlab for classical elliptic and parabolic linear equations, as well as the evaluation of algorithmic aspects, accuracy, stability and efficiency.
- Autonomy of judgment in evaluating the approximation algorithms and the obtained results also through discussion with one's peers in possible team work.
- Ability to clearly communicate the acquired concepts and to discuss the obtained results.
- Ability to learn the drawbacks and the advantages of models and methods of resolution and to apply them in different working and scientific contexts.

Prerequisites:

- Basic methods and algorithms of numerical analysis.
- Knowledge of a programming language.

Course contents summary:

- Relevant background in analysis: Sobolev spaces, variational formulations of elliptic PDEs, relevant functional analysis.
- Finite difference methods for elliptic problems: introduction, implementation, basic analysis.
- Galerkin methods for elliptic problems: stability, error analysis, implementation of standard finite element methods.
- Spectral methods for elliptic problems: spectral Galerkin and collocation methods.
- Methods for parabolic problems: time discretization, implicit and explicit Euler method.
- Advanced topics, including boundary element methods and adaptive methods.

Recommended readings:

- "Finite Elements", D. Braess, Cambridge University Press, 2010.
- "Numerical Approximation of Partial Differential Equations", A. Quarteroni, A. Valli, ed. Springer, 1994.
- "Spectral Methods: Algorithms, Analysis and Applications", J. Shen, T. Tang, L.-L. Wang, Springer, 2011.
- "A finite element primer", D. J. Silvester, https://personalpages.manchester.ac.uk/staff/david.silvester/primer.pdf .
- "Spectral Methods in Matlab", L. N. Trefethen, SIAM, 2000.

Teaching methods:

During the lectures the contents of the course will be analyzed, highlighting the difficulties related to the introduced numerical techniques. Moreover, the course will consist of a part of autonomous re-elaboration, supervised by the professor, consisting in the application of the numerical techniques through laboratory programming. This activity will allow students to acquire the ability to deal with "numerical" difficulties and to evaluate the reliability and consistency of the obtained results.

Assessment methods and criteria:

The exam includes:

- the assignment of a work for the application of numerical techniques introduced to solve a specific problem. The analysis of the results obtained by the student will allow to evaluate the acquisition of the above listed skills. In particular the threshold of sufficiency is fixed to the ability to achieve reliable numerical results.
- an assessment of the knowledge through a discussion of topics of the course. The threshold of sufficiency consists in the knowledge of the discriminating characteristics of the various methods presented in the course.